Noise-induced traveling waves in electroconvection

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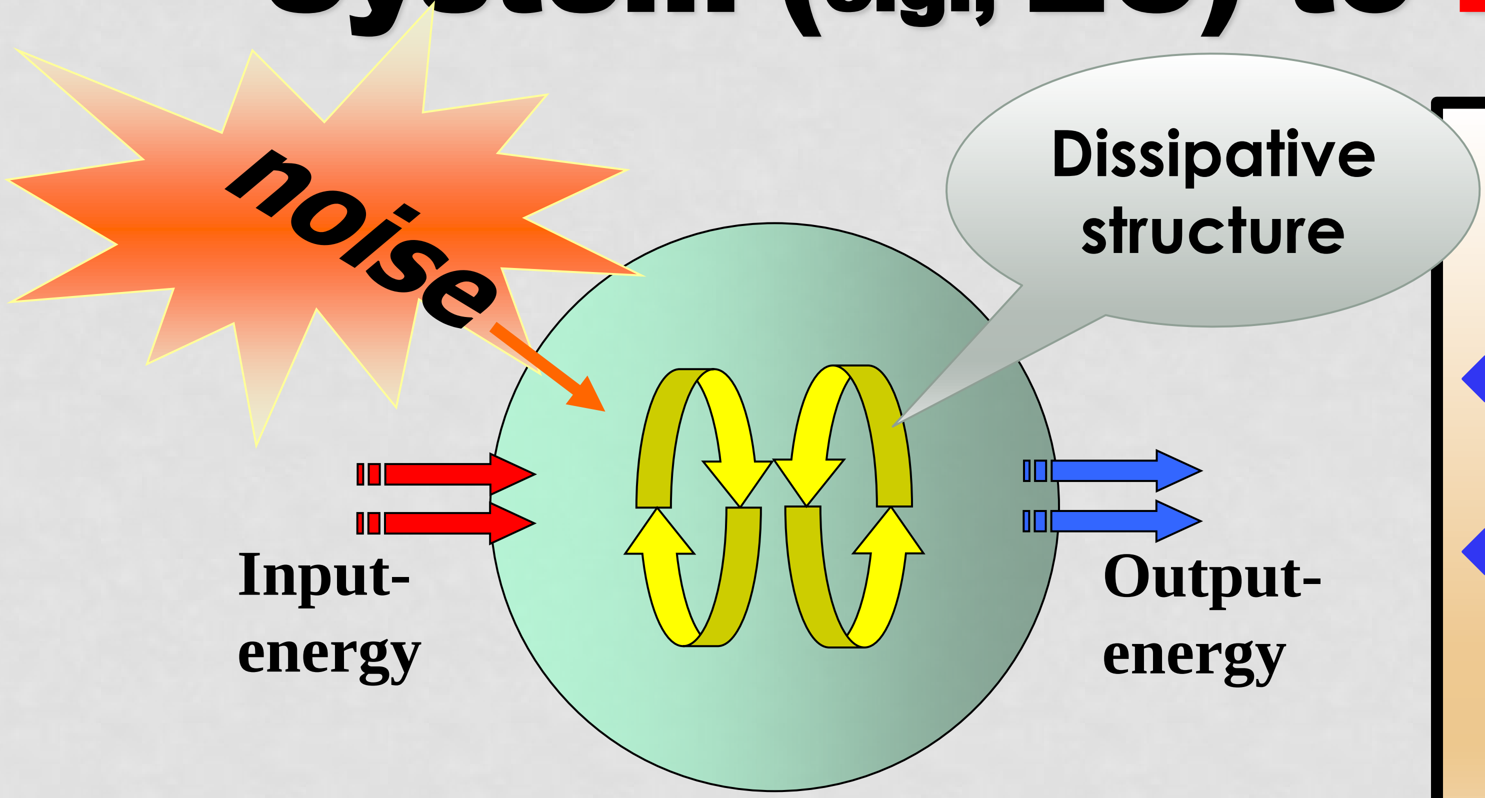
Response of Dynamical Dissipative System (e.g., EC) to External Noise

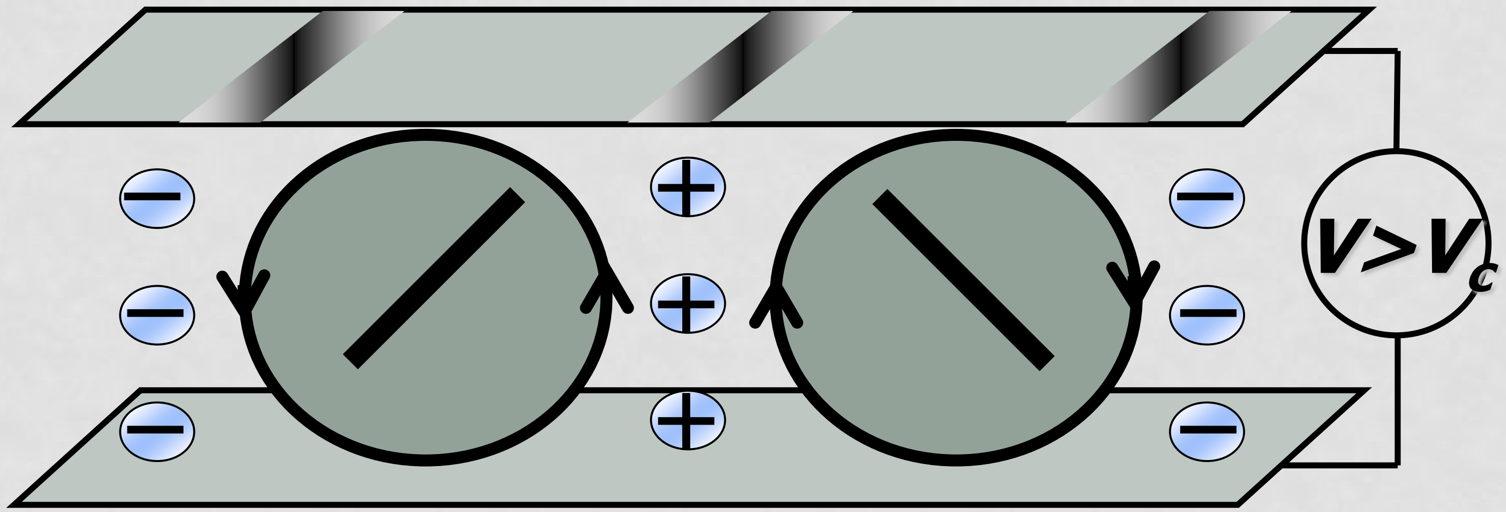


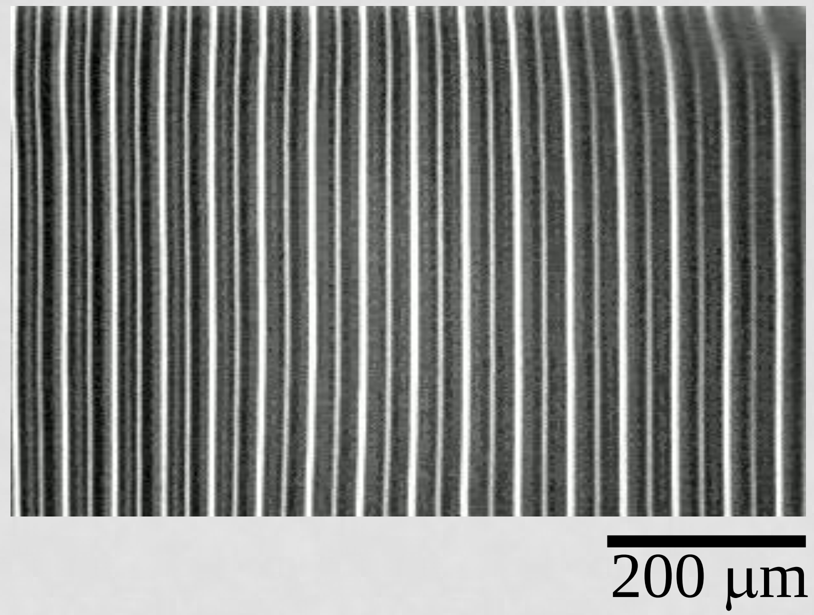
Nonequilibrium Systems

Our aim is to examine:

- **Hopf frequency of TW**
- **Pattern formations**

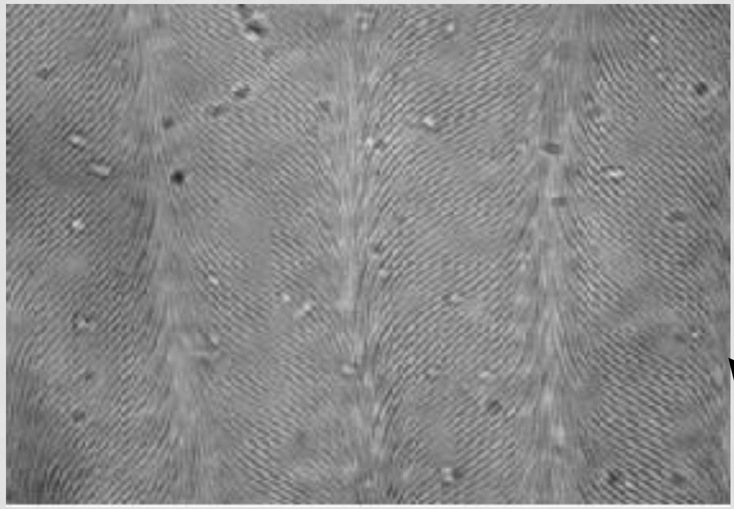
(in the presence of noise)



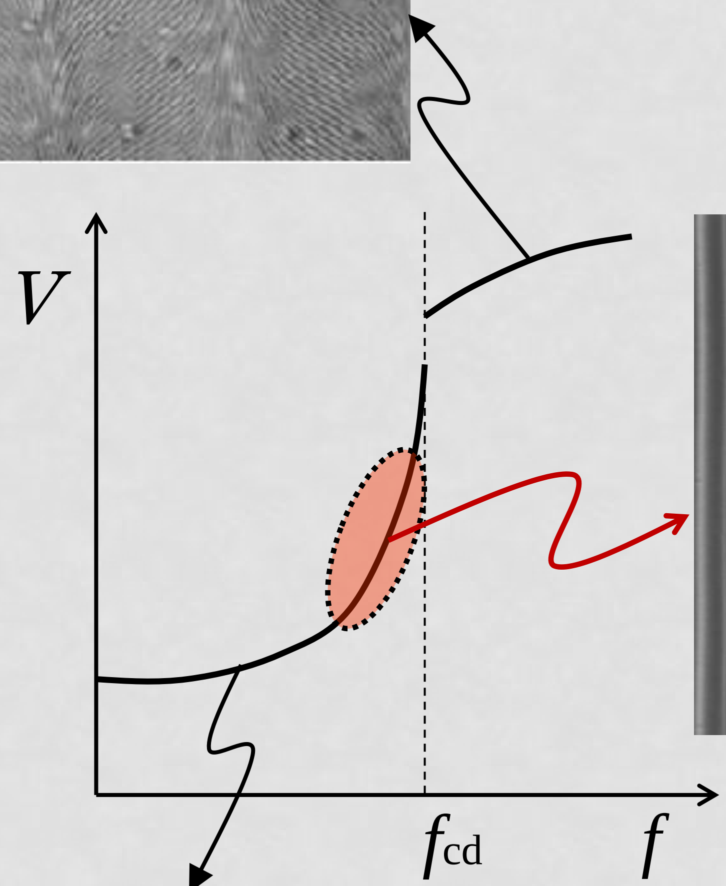


Carr-Helfrich mechanism for ac-driven Electroconvection (EC) induced in nematic liquid crystals and a typical EC pattern (Williams domain)

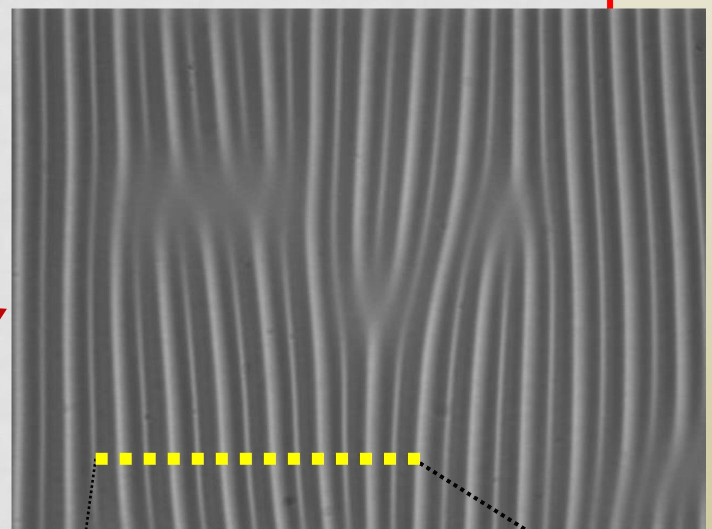
What is Traveling wave (TW) ?

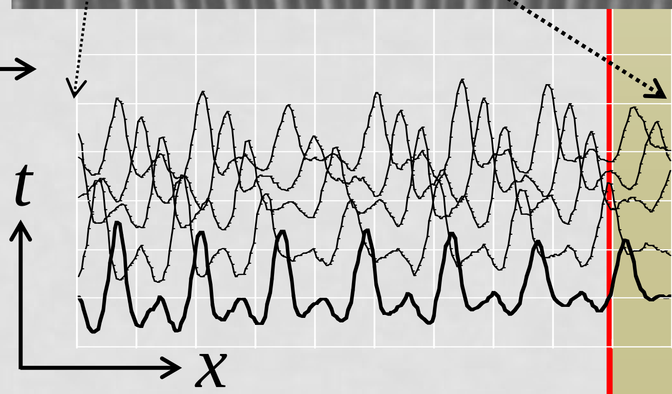


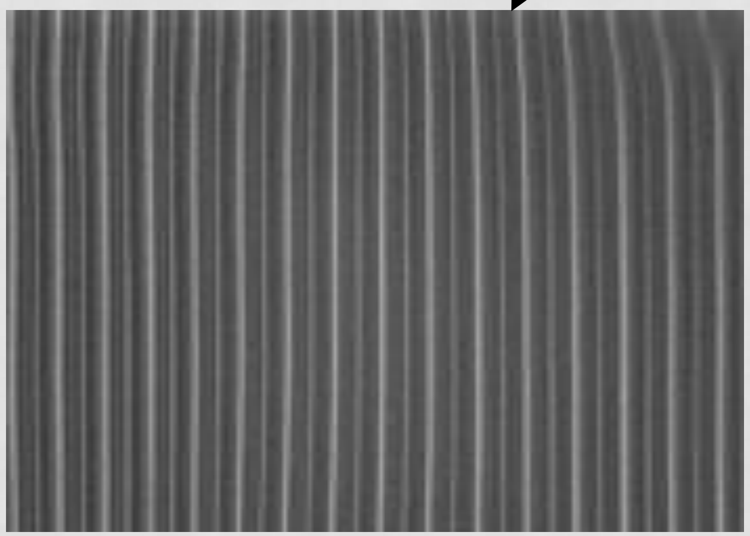
Chevron (CV)



Traveling Wave (TW)







Williams Domain (WD)

▸Weak-electrolyte model (WEM) for TW

$$\dot{A}_\sigma = \lambda_\sigma(R)A_\sigma - \tilde{\alpha}^2 R\sigma_a^{eff}(\sigma_\perp\tau_d)^{-1}A_n$$

$$\dot{A}_n = \frac{R\sigma_\perp}{\sigma_a^{eff}\tau_d}\left(\frac{C}{1+(\beta\Omega\tau_q)}\right)^2 A_\sigma + \lambda_n(R)A_n$$

M. Dennin, M. Treiber and L. Kramer, G. Ahlers, and D. S. Cannell, Phys. Rev. Lett. 76, 319 (1996).

$$\boxed{\begin{aligned} \frac{dA_\sigma}{dt} &= f(A_\sigma, A_n, R), \\ \frac{dA_n}{dt} &= g(A_\sigma, A_n, R) \end{aligned}}$$

The WEM can be considered as an activator-inhibitor model for a Hopf bifurcation (if $\partial f/\partial A_n < 0,\ \partial g/\partial A_\sigma > 0$).

A_σ : **the amplitude of the local deviation of the conductivity**

A_n : **the director amplitude**

$R(=V^2/V_c^2)$: **a control parameter**

Experiment

◆ Important Parameters

☑ EC in Liquid Crystals

▷ $\tau_{\mathrm{d}} = \dfrac{\gamma_1 d^2}{K_{33}} \sim 10^{-1}\ \mathrm{s}$

: the director relaxation time

▷ $\tau_{\sigma} = \dfrac{\varepsilon_0 \varepsilon_{//}}{\sigma_{//}} \sim 10^{-3}\ \mathrm{s}$

: the charge relaxation time

▷ $f^{-1} = \dfrac{2\pi}{\omega} \sim 10^{-3}\text{–}10^{0}\ \mathrm{s}$

▷ $V \sim 0-10^2\ \mathrm{V}$

internal timescales

▷ $\tau_{\mathrm{rec}} = \dfrac{1}{2k_r n_0} \sim 10^{0}\ \mathrm{s}$

▷ $\tau_{\mathrm{mig}} = \dfrac{d^2 V^{(0)}}{\pi^2(\mu^+ - \mu^-)} \sim 10^{-2}\ \mathrm{s}$

additional timescales in the WEM

☑ External noise

by Ornetein-Uhlenbeck process

▲ $\langle \xi(t) \rangle = 0$

▲ $C(t-t') = \langle \xi(t)\xi(t') \rangle$

$$= \frac{Q}{\tau_N} \exp(-|t-t'|/\tau_N)$$

external timescale

▷ $\tau_{\mathrm{N}} = \dfrac{1}{2\pi f_{\mathrm{c}}} \sim 10^{-6}\text{–}10^{-3}\ \mathrm{s}$

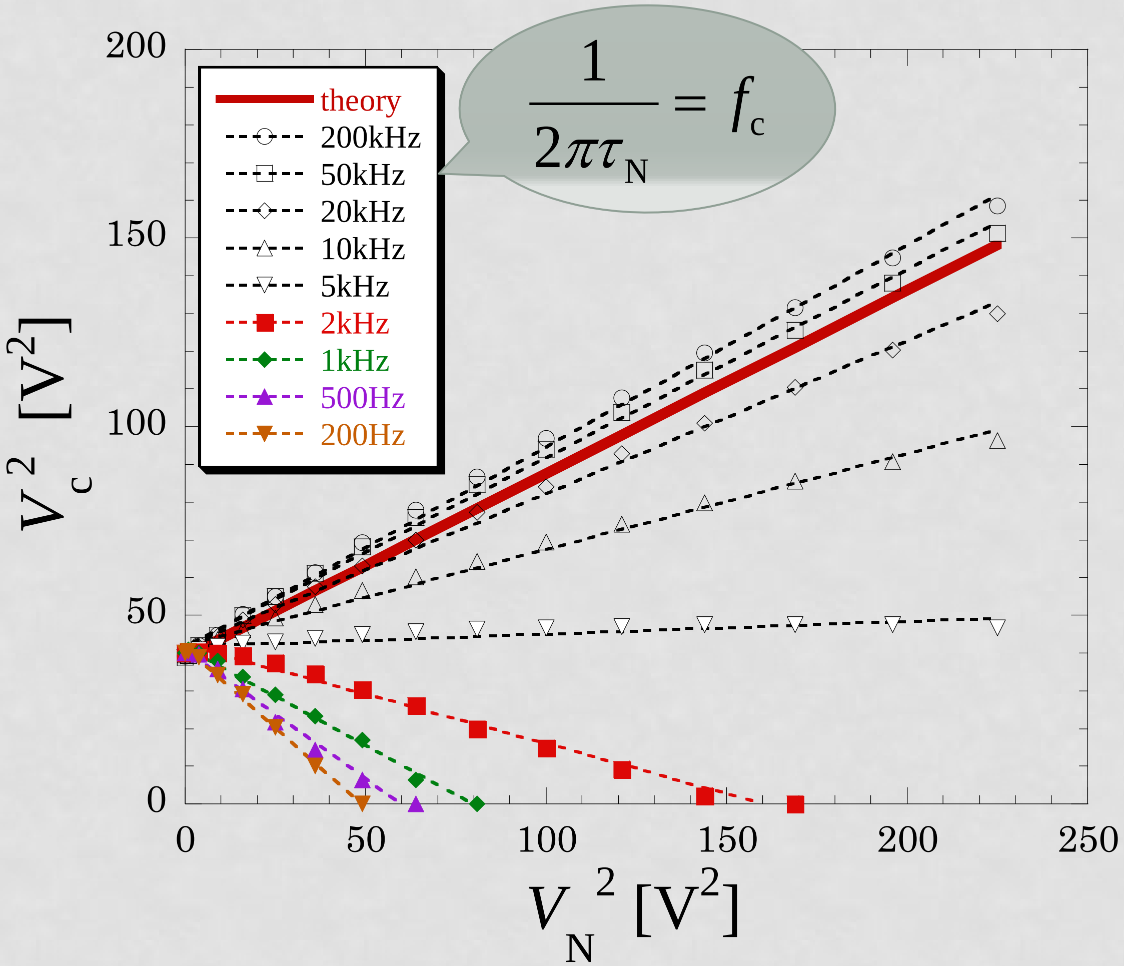
: the correlation time

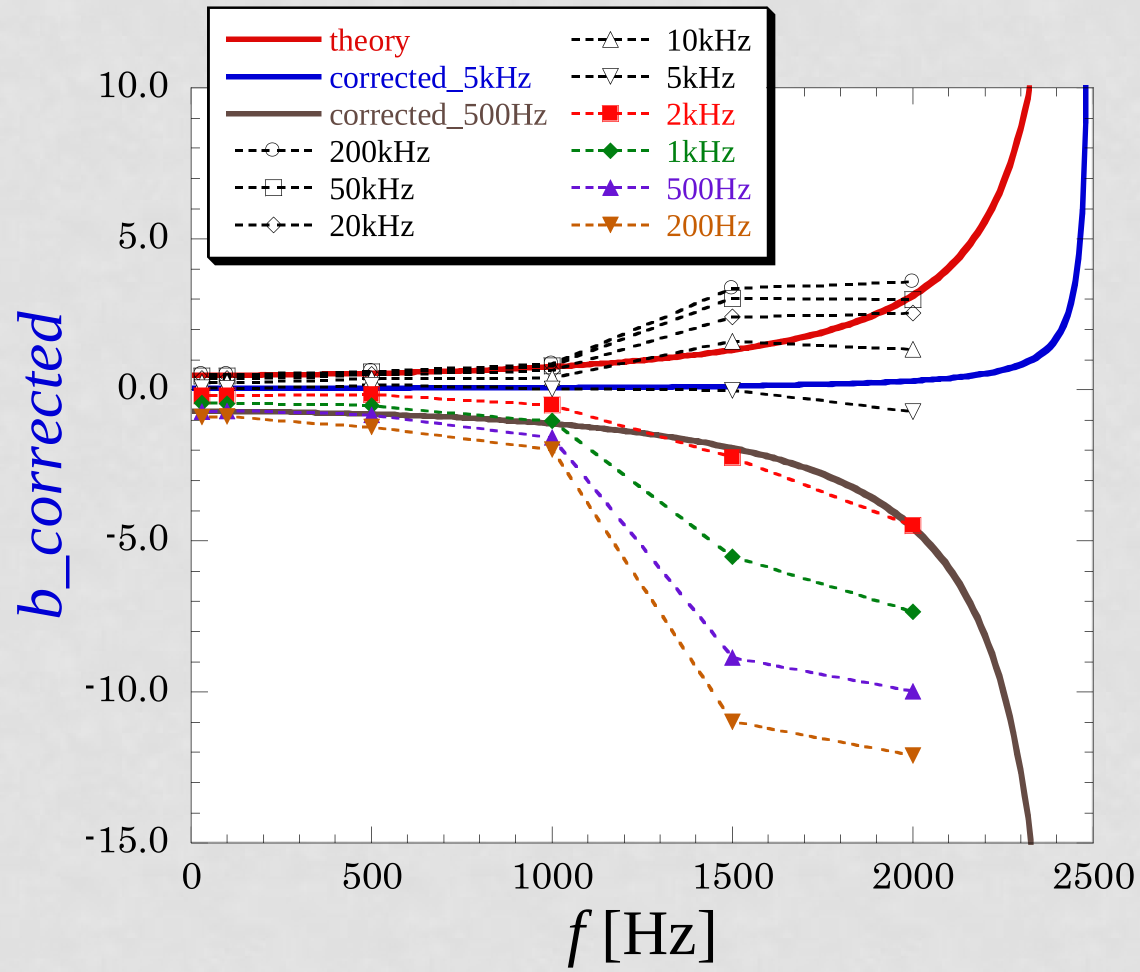
▷ $V_{\mathrm{N}} = d\sqrt{\langle \xi(t)\xi(t) \rangle} \sim 0-30\,\mathrm{V}$

: noise intensity

Previous study :noise-induced EC threshold problems

◆ Instability Problem (Threshold Vc of EC)





Theory

☑ Kawakubo 81
for b > 0 (white noise-limit)

$$\boxed{V_c^2 = V_{c0}^2 + bV_N^2} \quad \boxed{b = \frac{(1+\omega^2\tau_\sigma)}{\zeta^2 - (1+\omega^2\tau_\sigma)}}$$

Experiment

☑ Huh 2014
for b > 0, b < 0 (colored noise)

$$\boxed{V_c^2 = V_{c0}^2 + b_{cor}V_N^2, \quad b_{cor} = b\left(1 - h\frac{\tau_N}{\tau_\sigma^n}\right)}$$

Experimental results for TW (1)

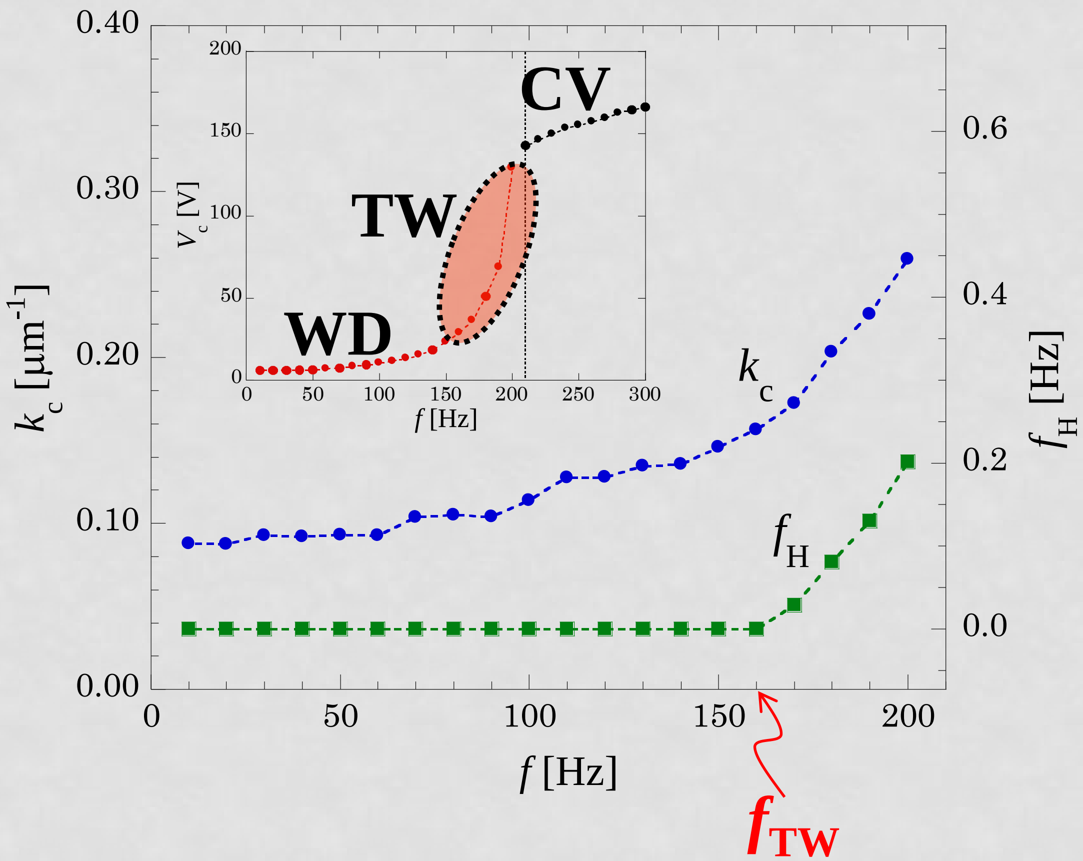
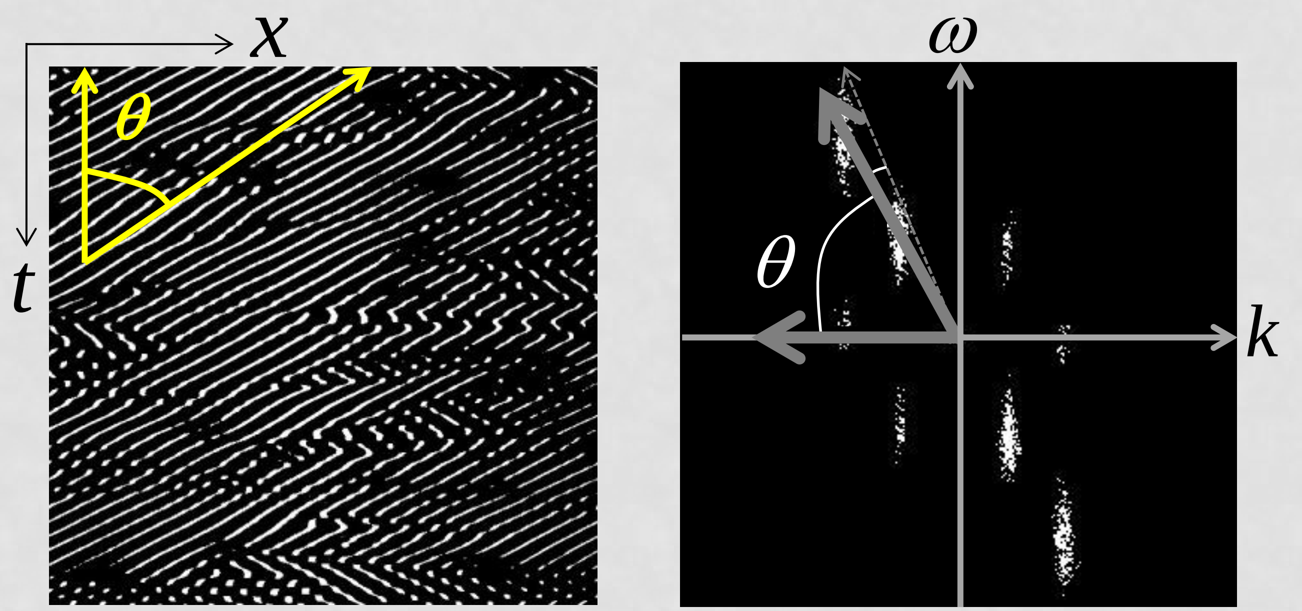


Fig.1 EC-threshold V_c, wave number k_c, and Hopf frequency f_H as functions of ac frequency f . For $f > f_{TW}$ (~160 Hz), TW arises as a primary instability.



Space-time map

$$\omega_c = 2\pi f_H = k_c \tan\theta$$

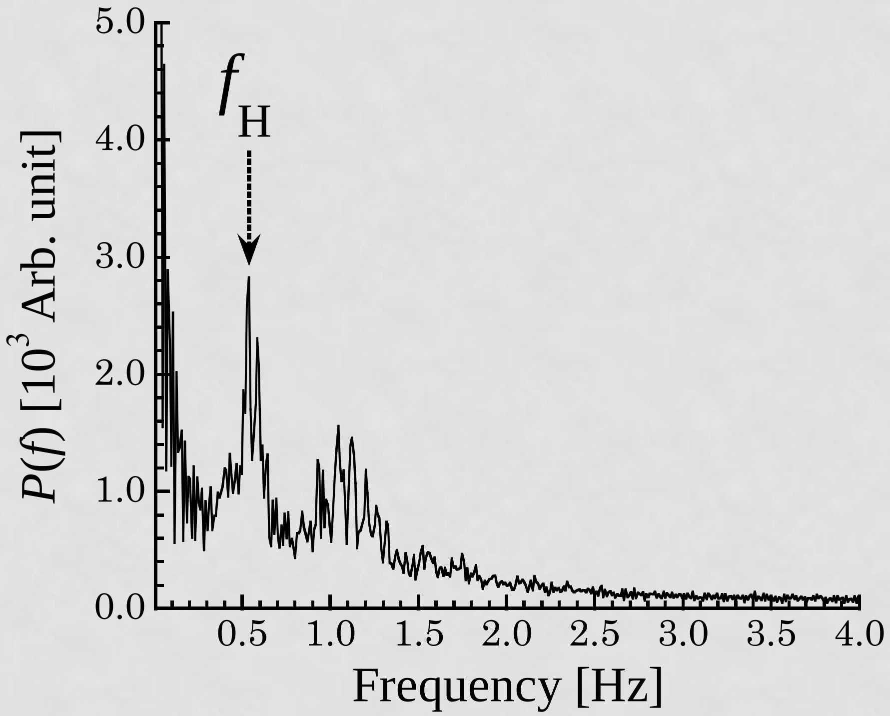


Fig.2 Measurement of Hopf frequency f_H. Space-time map was obtained by successive arrangement of a one-dimensional image arbitrarily selected in TW with an identical time interval.

"Traveling waves and localized waves of electroconvection in external multiplicative noise," Jong-Hoon Huh, PRE95, 042704 (2016).

Experimental results for TW (2)

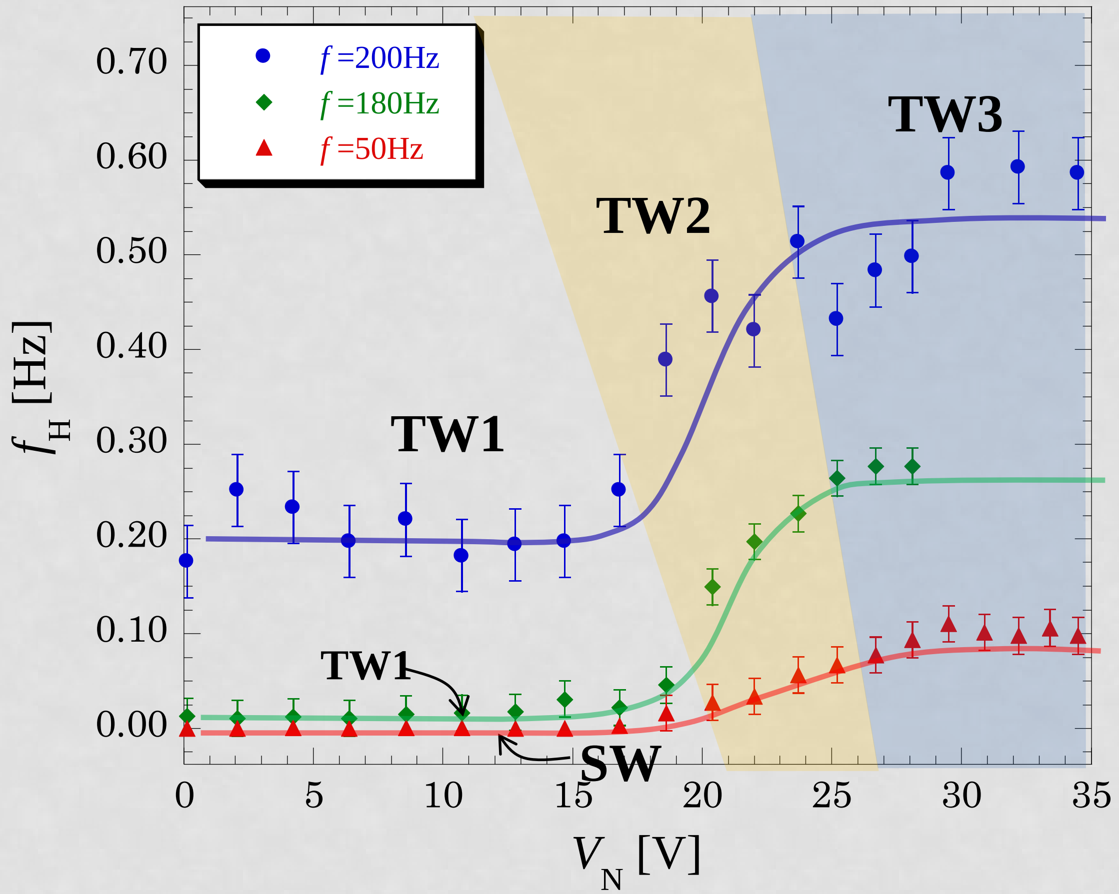


Fig.3 Noise-induced traveling waves. Increasing noise intensity V_N, unique pattern evolutions are observed: SW→TW1→TW2→TW3 (for $f < f_{TW}$), and TW1→TW2→TW3 (for $f > f_{TW}$).

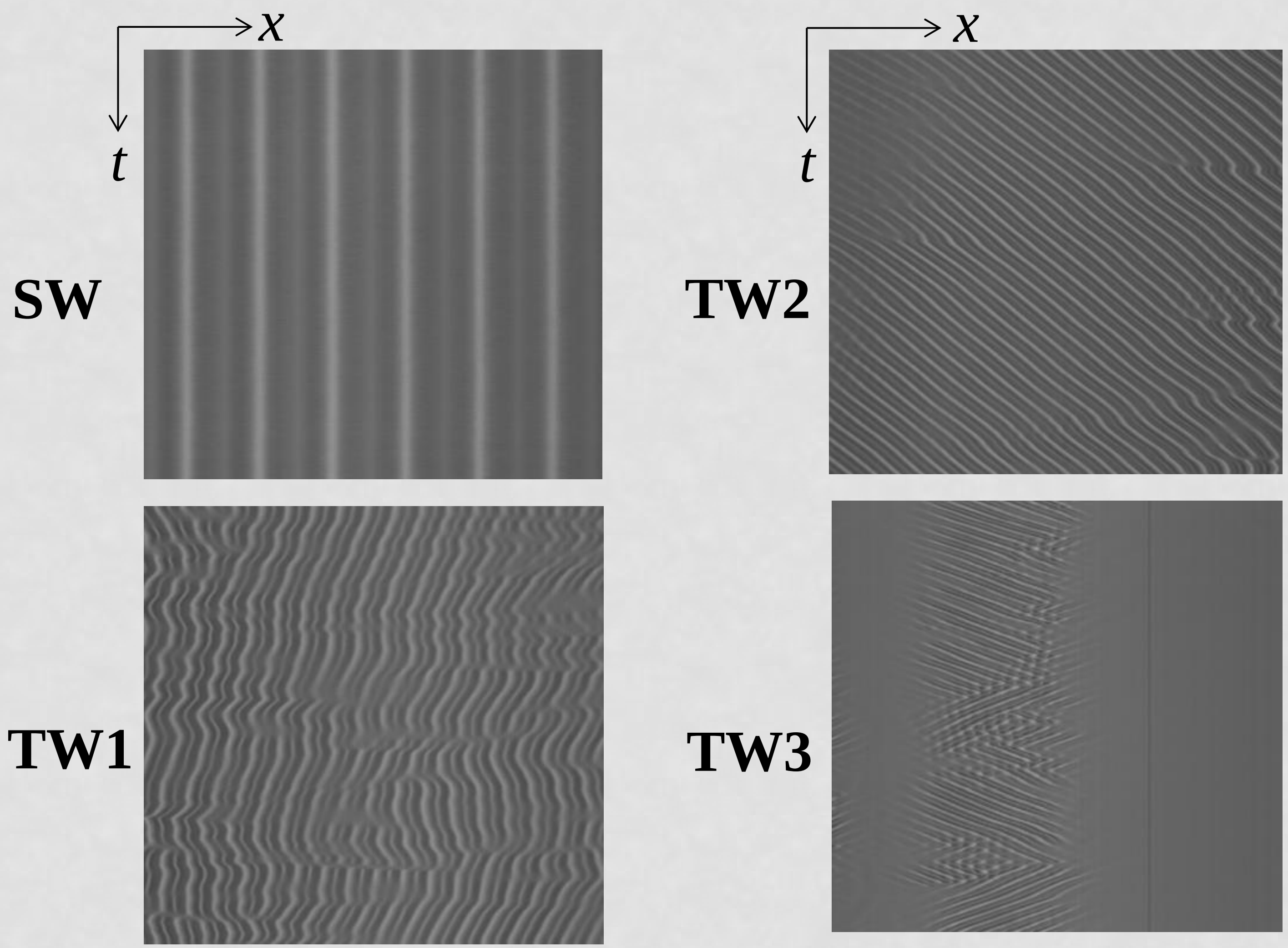


Fig.4 Noise-induced traveling waves: SW (stationary wave, $f_H = 0$), TW1 (traveling wave with defect motions), TW2 (without defects), TW3 (localized TW).

Experimental results for TW (3)

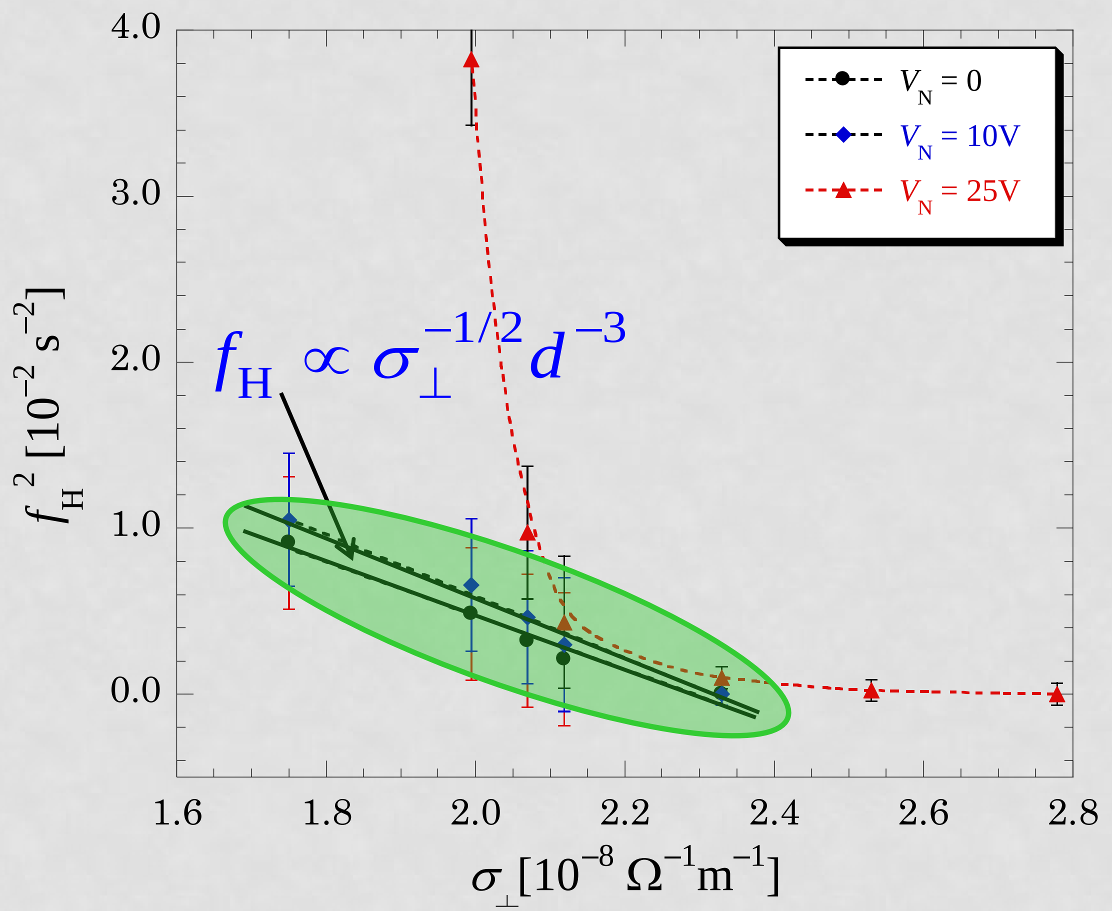


Fig.5 Comparison between the WEM prediction and experimental results (with different noise intensities).

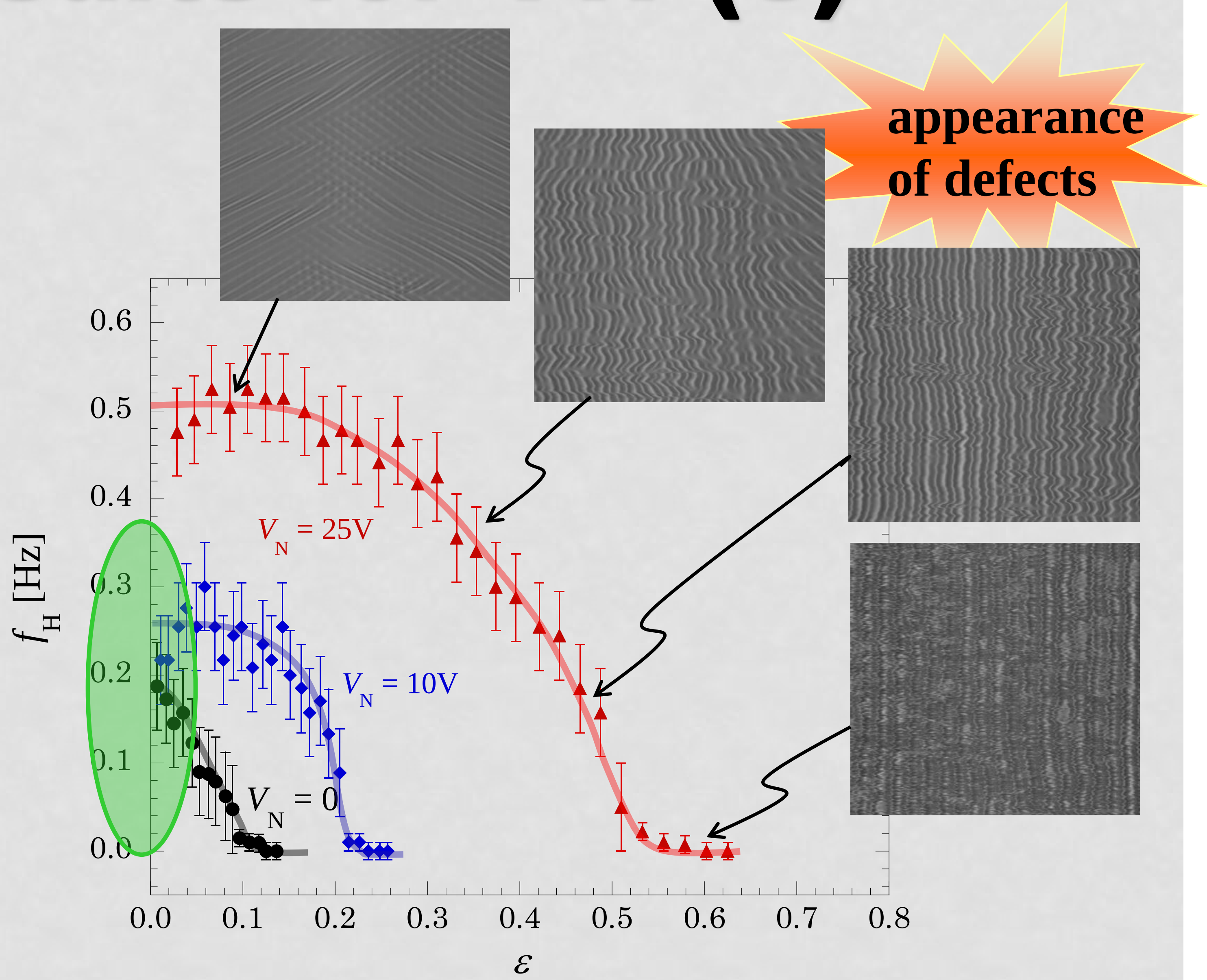


Fig.6 Dependence of Hopf frequency f_H on a reduced ac voltage $\varepsilon=(V^2-V_c^2)/V_c^2$ in the absence and presence of noise. TW ($f_H \neq 0$) smoothly changes into an SW ($f_H = 0$); however, this SW is completely different from the SW around $\varepsilon = 0$ (in Fig.4). See each space-time map for various ε.

- ☑ **The WEM works well at low V_N and low V. See green regions in Figs.5 and 6.**
- ☑ **The WEM is inapplicable to high V_N (due to unknown noise effects) and high V (due to defects).**

Discussions and Summary (1)

- **For low noise intensities ($V_N < 15$ V), the WEM for TW works well.**
  → **See Fig.1 and Fig.5.**
- **For high noise intensities ($V_N > 15$ V), the WEM is inapplicable to TW.**
  → **See Fig.3 and Fig.5.**
- ***V*-dependent Hopf frequency shows unexpected behavior (Fig.6) due to the motion of defects; the role of *V* should be examine in defect-free EC.**
  → **See the proceeding of ICNF2017, and "Electroconvection in in-plane switching-mode cells" (submitted to PRE.**

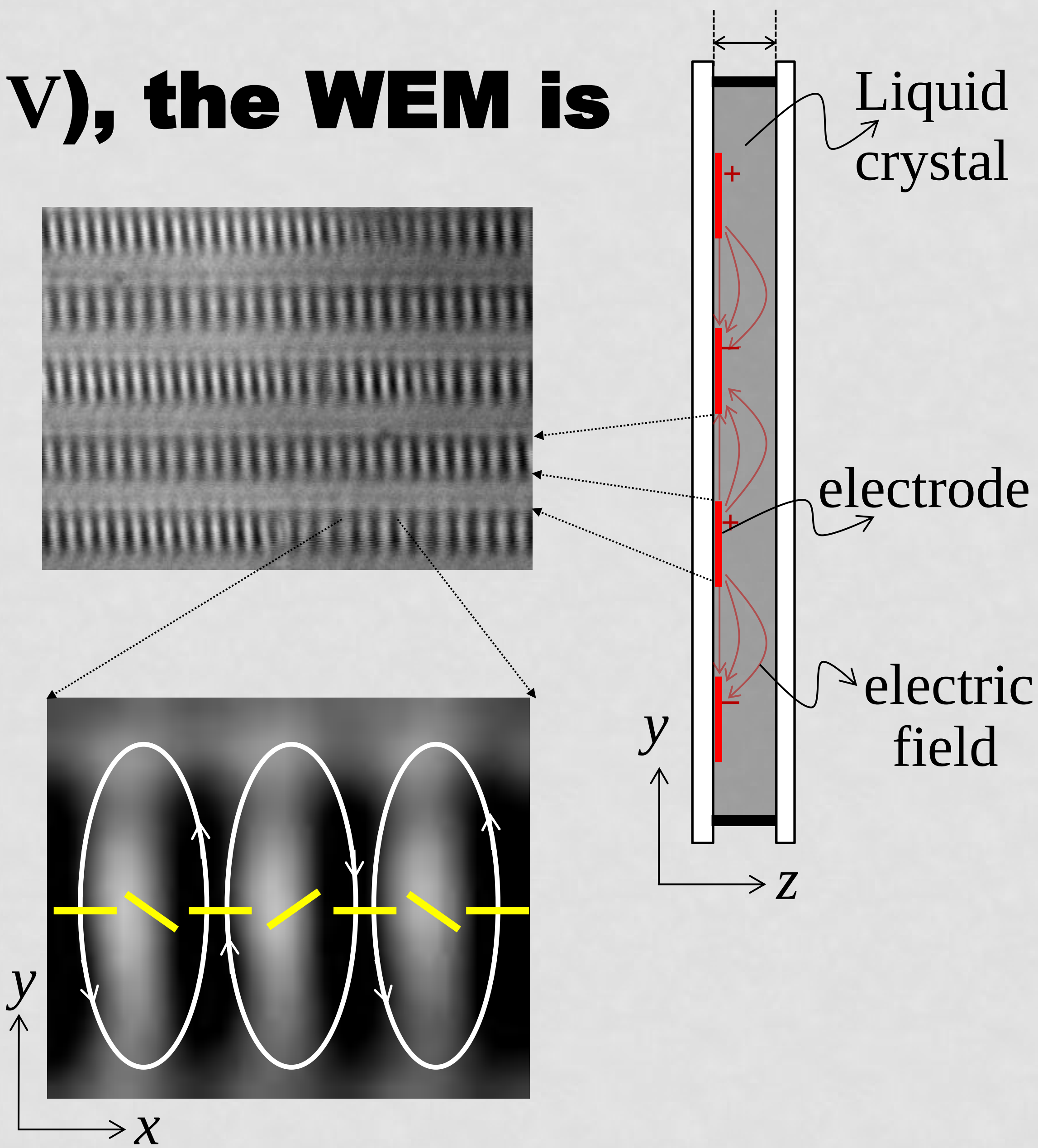


Fig.7 Defect-free EC in quasi-one dimensional cells prepared by employing the in-plane switching mode.

Discussions and Summary (2)

- **What happens in the WEM under external noise ?**

$$\dot{A}_\sigma = \lambda_\sigma(R)A_\sigma - \tilde{\alpha}^2 R\sigma_a^{eff}(\sigma_\perp \tau_d)^{-1} A_n$$

$$\dot{A}_n = \frac{R\sigma_\perp}{\sigma_a^{eff}\tau_d}\left(\frac{C}{1+(\beta\Omega\tau_q)}\right)^2 A_\sigma + \lambda_n(R)A_n$$

←Noise may activate conductivity (σ)-related parameters (e.g., $\lambda_\sigma, \sigma_\perp, \sigma_a^{eff}$) due to noise-driven random oscillation of space charges in EC.

- **Colored noise can change the present results?**

e.g., $V-$ and V_N-dependent Hopf frequencies ?
(To understand this approach, see the previous study)

Our future study

Acknowledgement

This study was supported by JSPS KAKENHI (Grant No. 15K05215).

Pattern formations in electroconvection by colored noise

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Introduction① Mechanism of Electroconvection (EC)

Q. What's Liquid Crystals (LCs)?

A. ▪ state between liquid and solid crystals

▪ fluidity of liquid and anisotropy of solid crystals

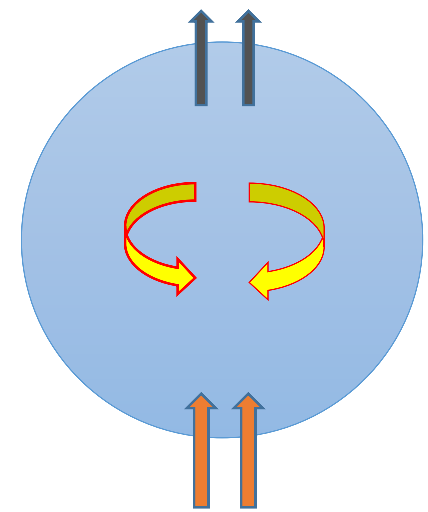
Q. What is the merits of EC?

A. ▪ easy to visualize

▪ easy to control (by electric parameter)

▪ efficiency of experiments (fast response)

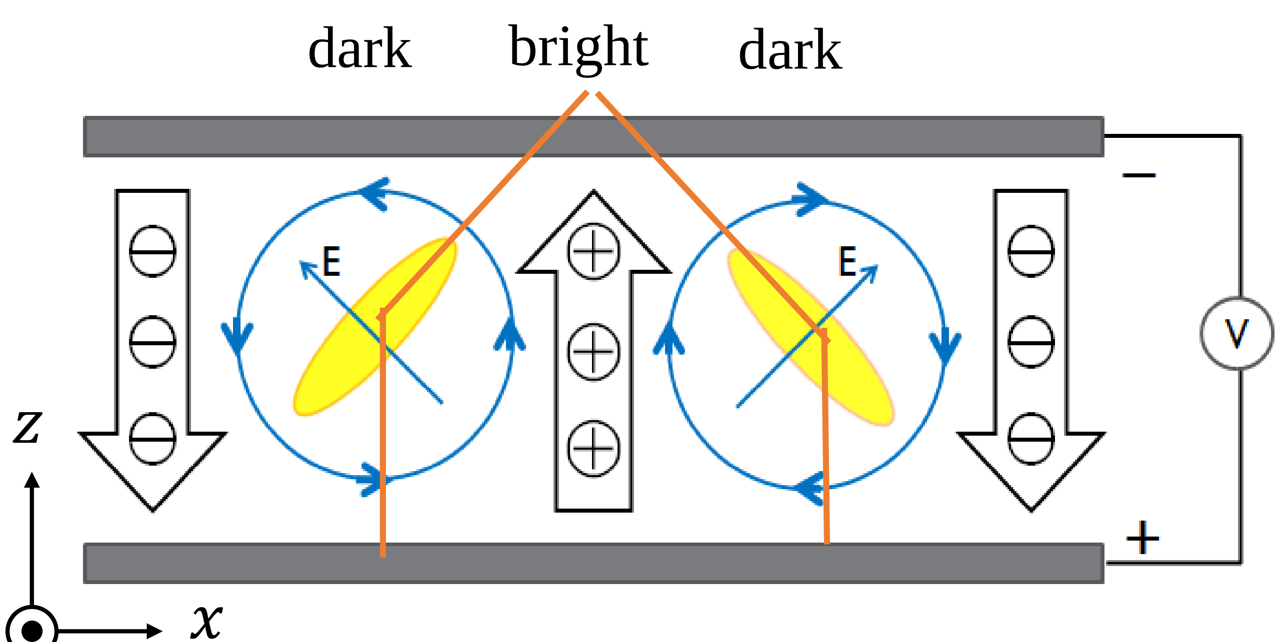
▪ abundance of convection pattern (due to anisotropy)



Nonequilibrium system

Internal Convention

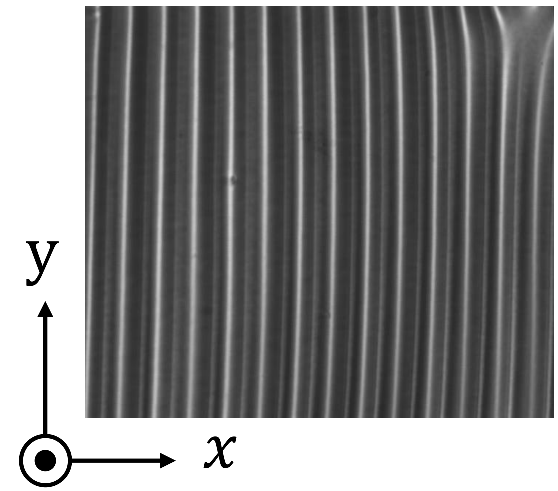
Input ≠ Output



Carr-Helfrich (1969)

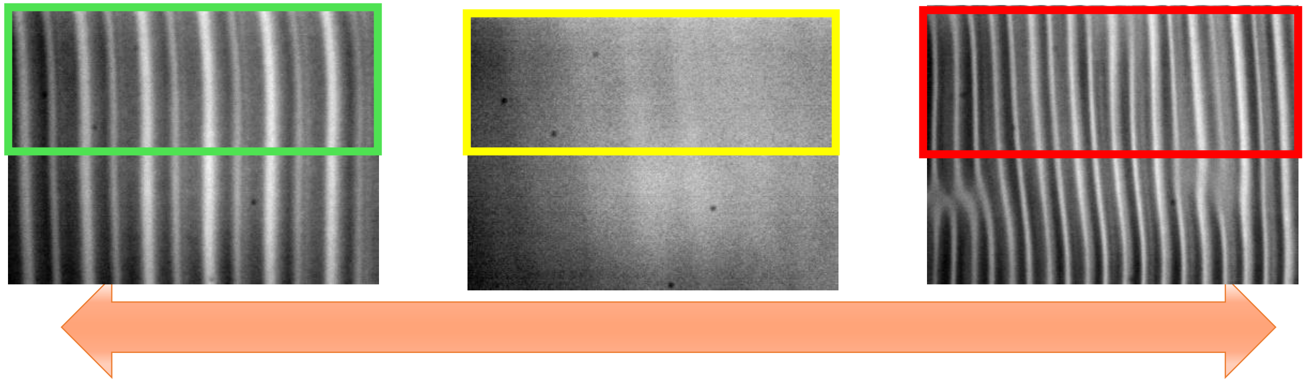
If voltage excess thresholds,

Electroconvection is occurred



Williams domain (1963)

Introduction② Spatio-Temporal Plot (STP) and Experimental Setup



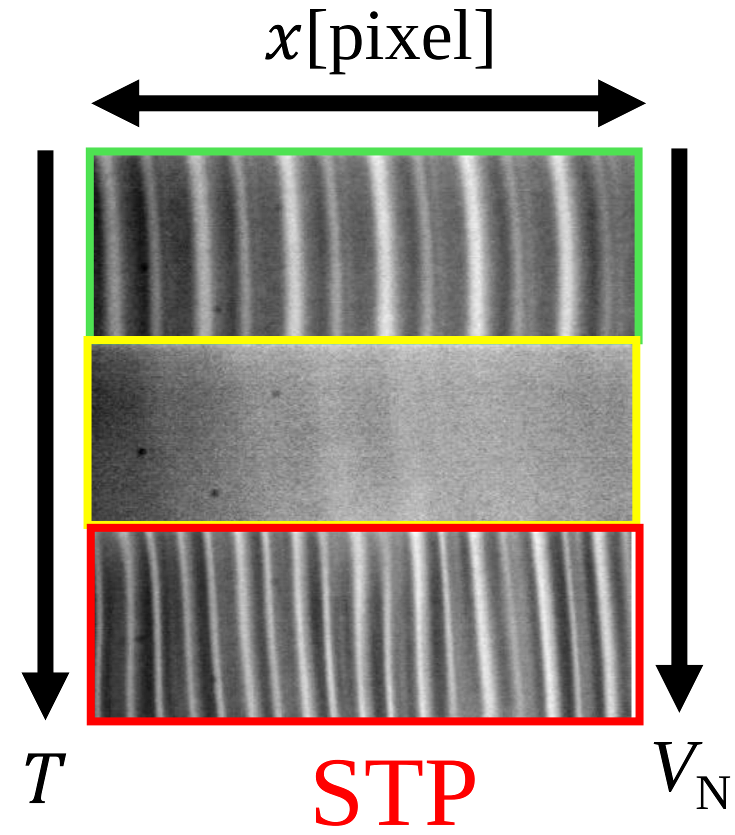
small **Noise Intensity V_N** large

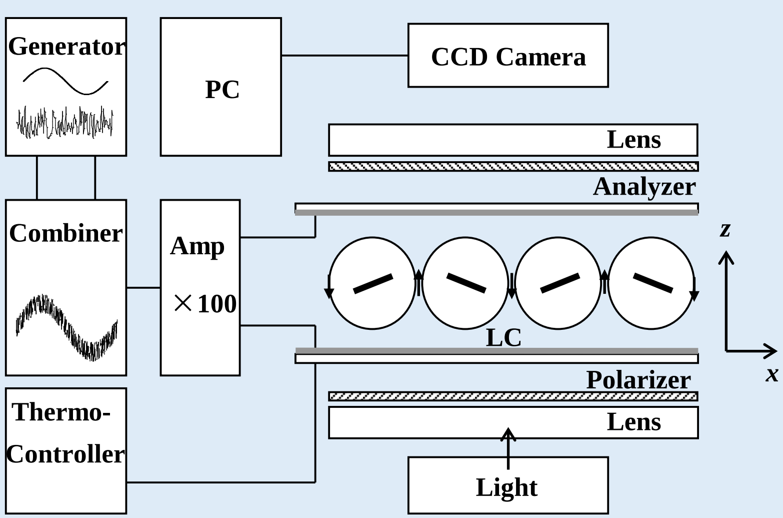
external multiplicative noise $N(t)$

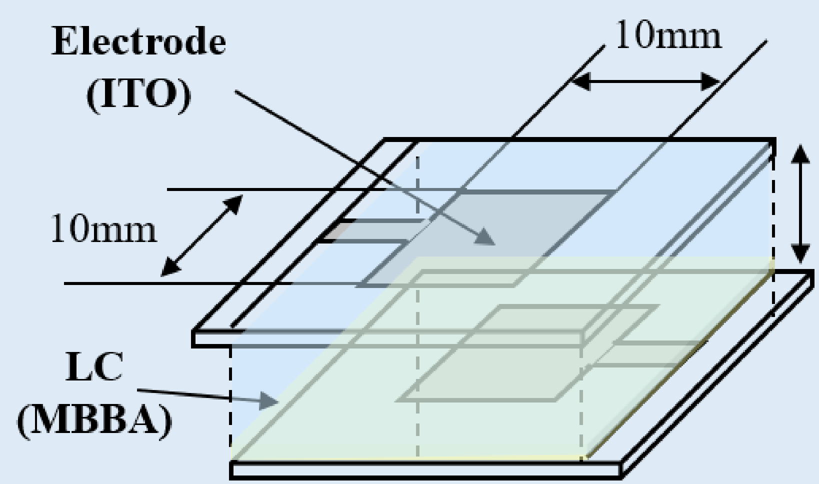
Nosie intensity: $V_N = d\sqrt{N^2(t)}$ [V]

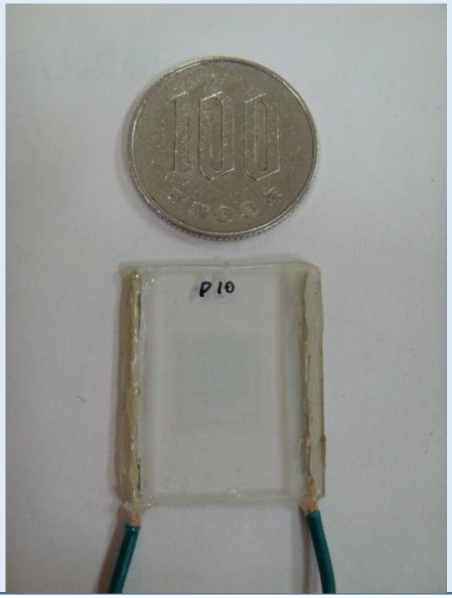
(Here, d is the thickness of sample cells)

T : measurement time[s]

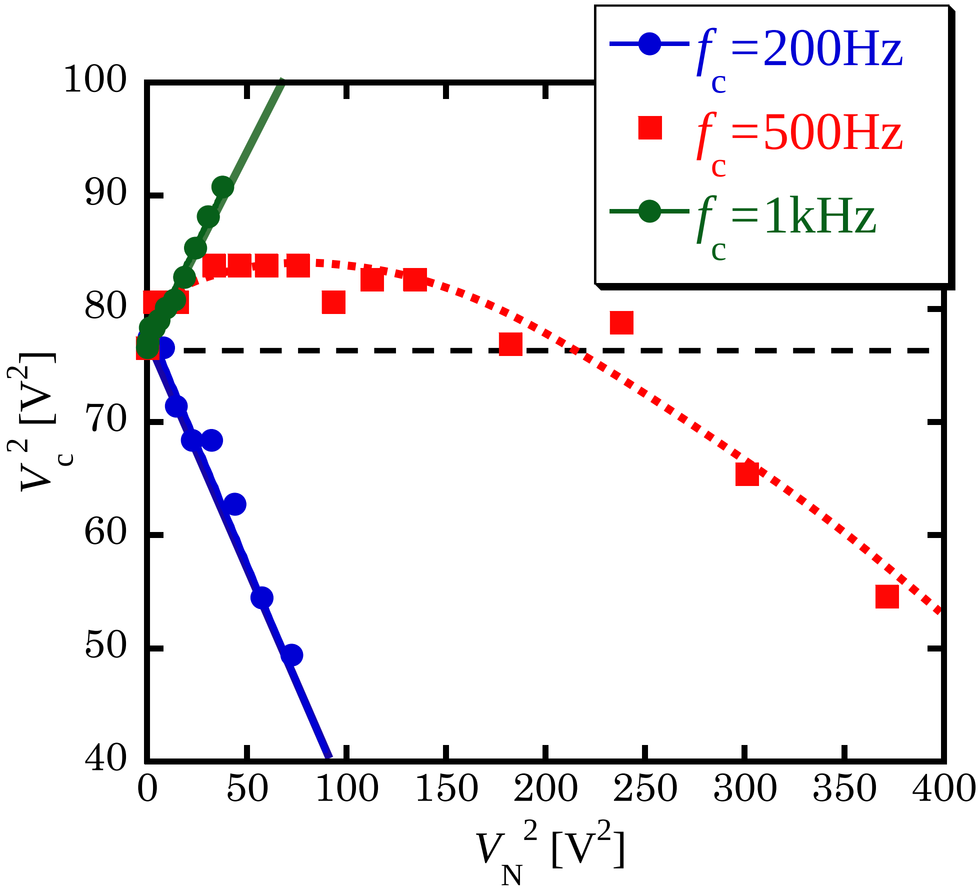








Experimental Results ① Measurement of Threshold by Colored Noise



J. -H. Huh, Phys. Rev. E 94, 052702 (2016).

f_c : cutoff frequency

f_c=200Hz
→V_c is monotonically decreased (b < 0)
noise effect → destabilizer

f_c=1kHz
→ V_c is monotonically increased (b > 0)
noise effect → stabilizer

f_c=500Hz
→V_c behavior cannot be fitted
by the theoretical formula below.

Carr-Helfrich mechanism considering white noise (f_c →∞, b > 0)
But we used "Colored Noise".

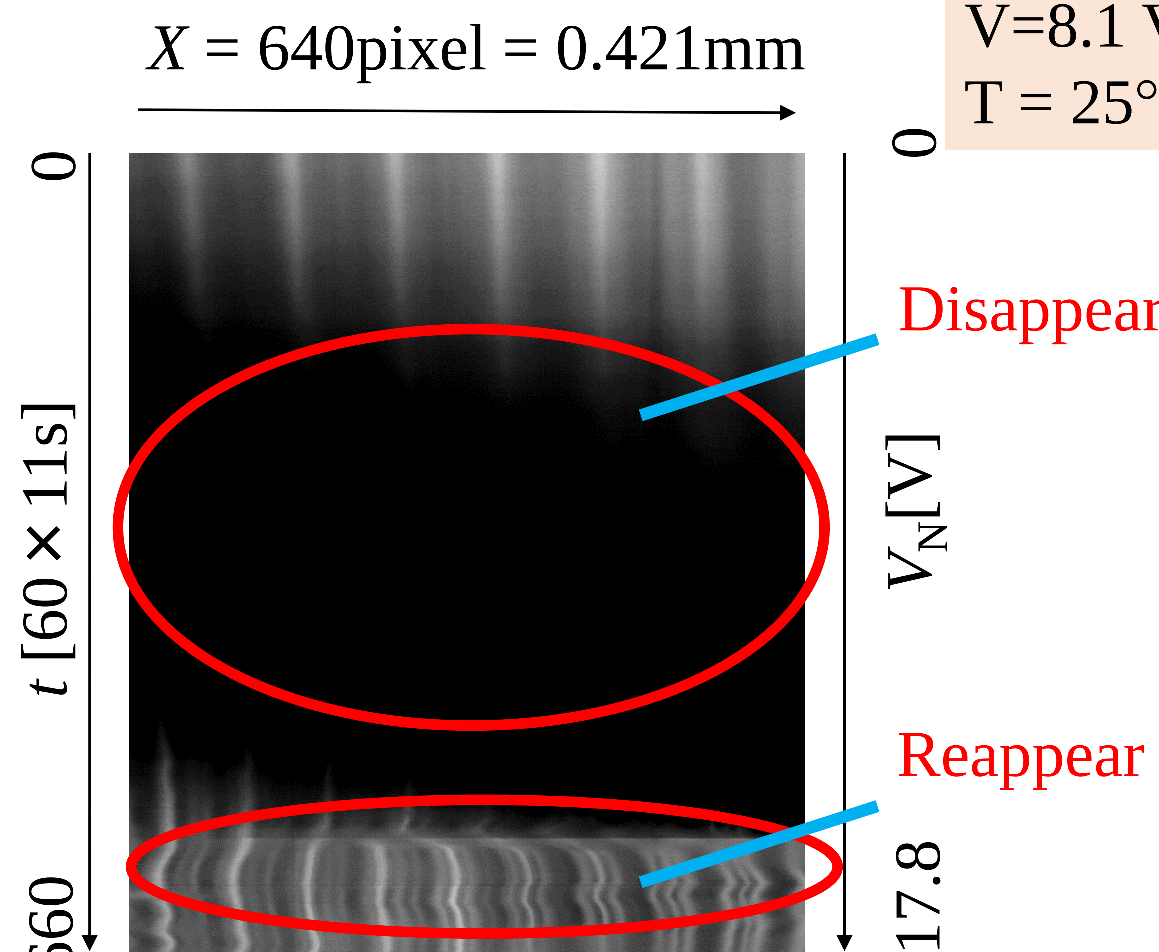
$$\boxed{\frac{1}{2}E_{MC}^2} = \boxed{\frac{\lambda\eta E_0^2(1+\omega^2\tau^2)}{\tau\sigma_H - \lambda\eta(1+\omega\tau^2)}} + \boxed{\frac{\lambda\eta E_0^2(1+\omega^2\tau^2)}{\tau\sigma_H - \lambda\eta(1+\omega^2\tau^2)}}\boxed{<\xi^2>_t}$$

$$V_c^2 = V_0^2 + bV_N^2$$

T. Kawakubo, A. Yanagita, and S. Kabashima, J. Phys. Soc. Jpn.**50**, 1451 (1981).

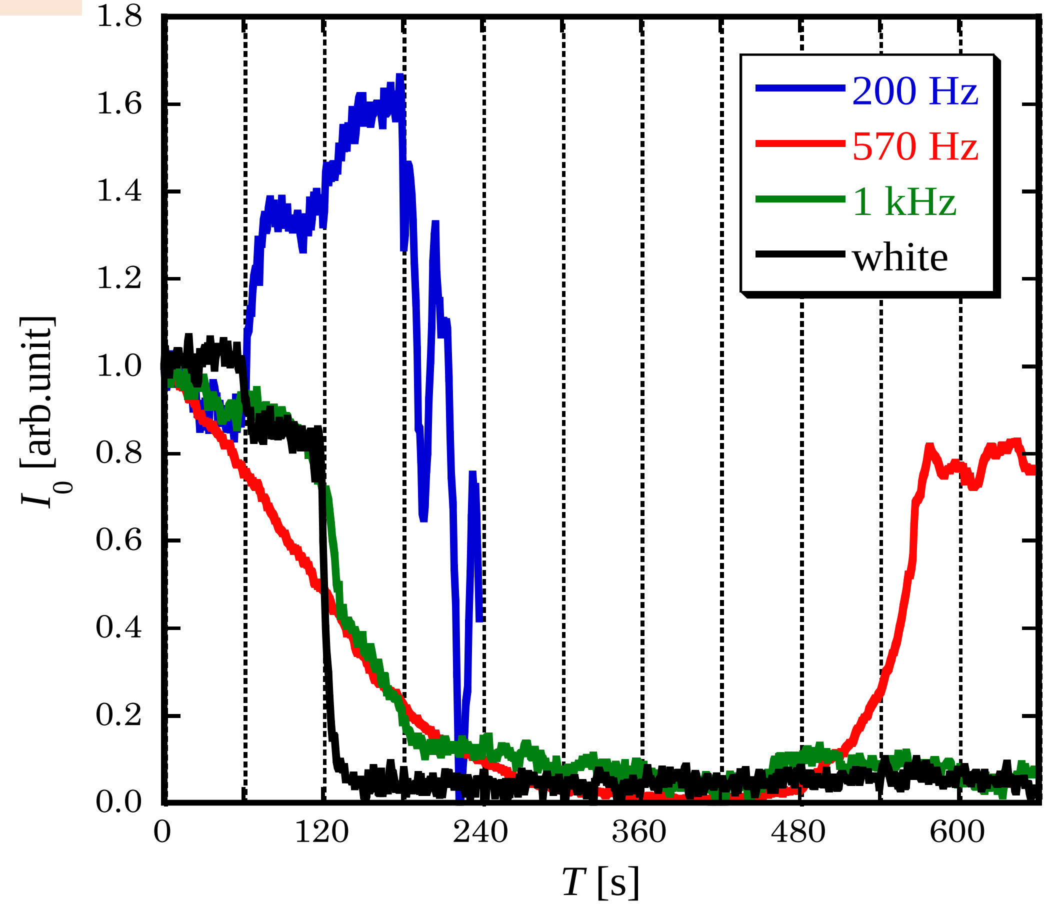
Experimental Results ② Measurement of Light Intensity

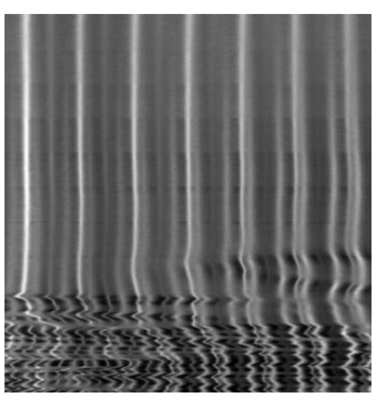
V=8.1 V
T = 25°C



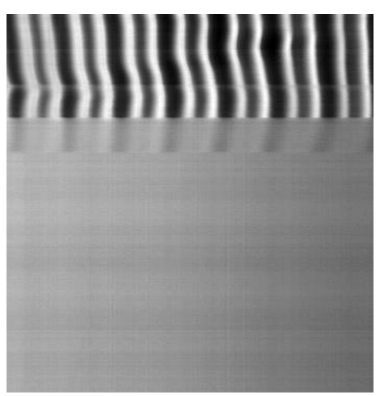
f_c=570Hz, 1.78V per 60s (step increase)

Light Intensity

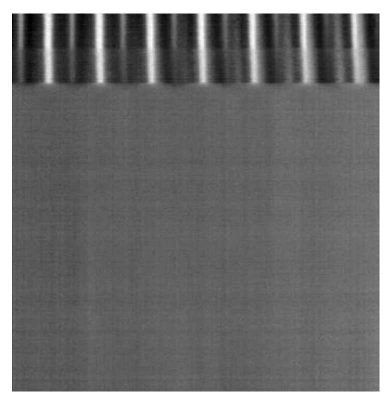




f_c=200Hz



f_c=1kHz

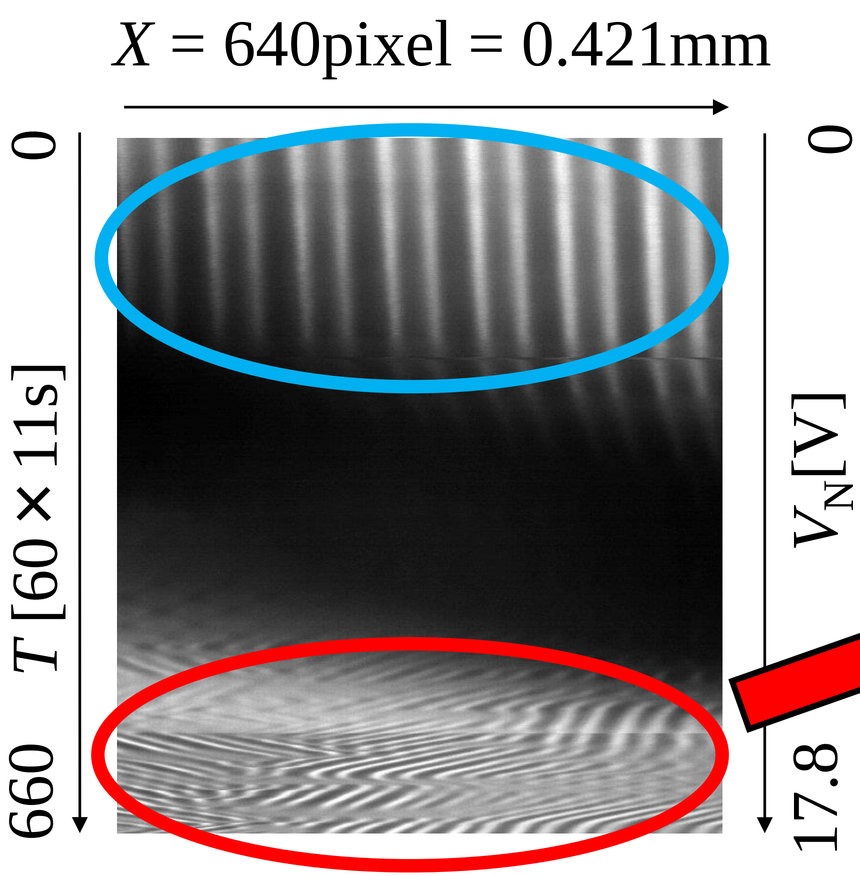


white noise

Light Intensity is …
f_c > 570Hz ➡ weakened
f_c < 570Hz ➡ strengthened

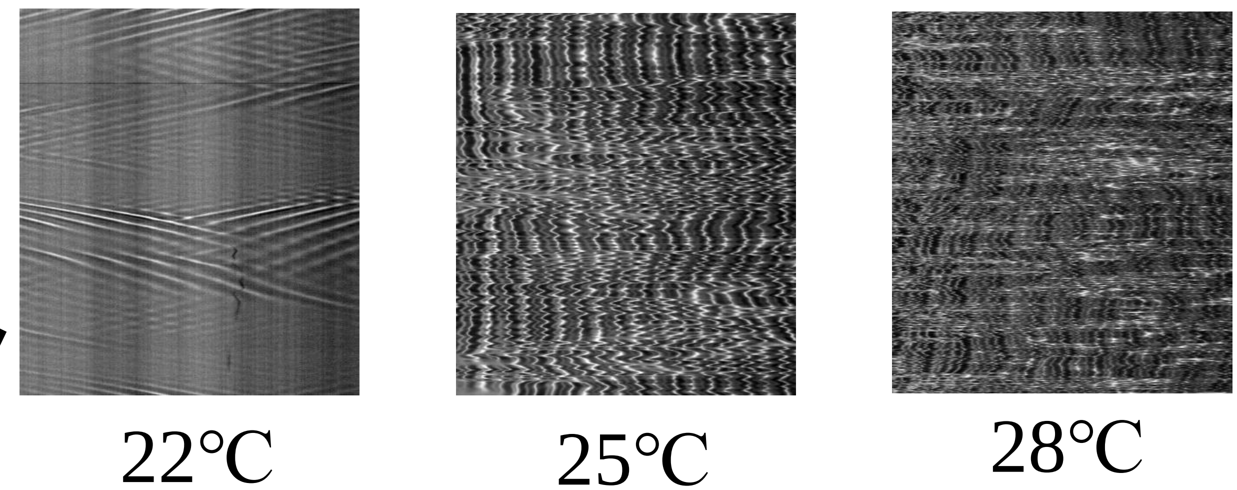
Inverse Stochastic Resonance

Experimental Results ③ Depending on Temperature on STP



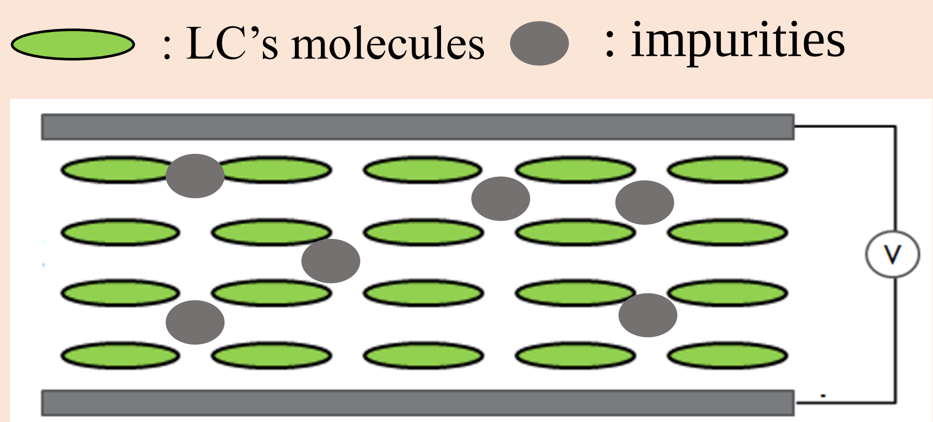
(f_c=570Hz, 22°C)

In a lower temperature,
Different pattern is observed.
It is called "Traveling Wave (TW)"



Pattern formations depending on temperature
V = 6.68V, V_N = 17.8V, f_c = 570Hz, for 660 s

Weak Electrolyte Model (WEM)



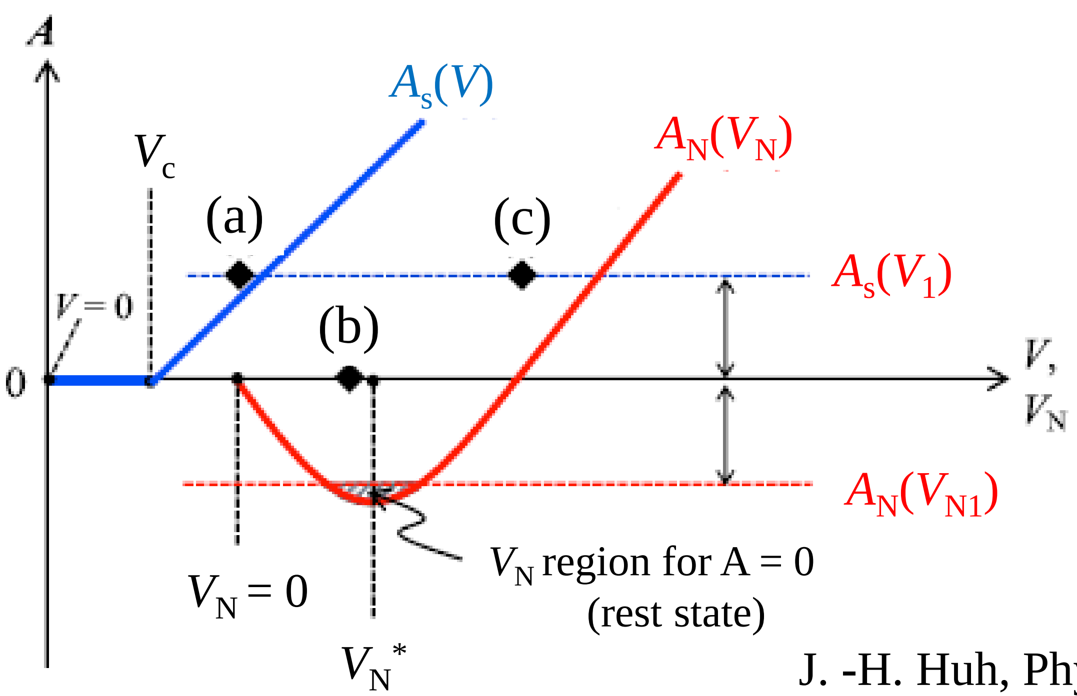
Taking dissociation and recombination into consideration
Also, impurities and dopants are included

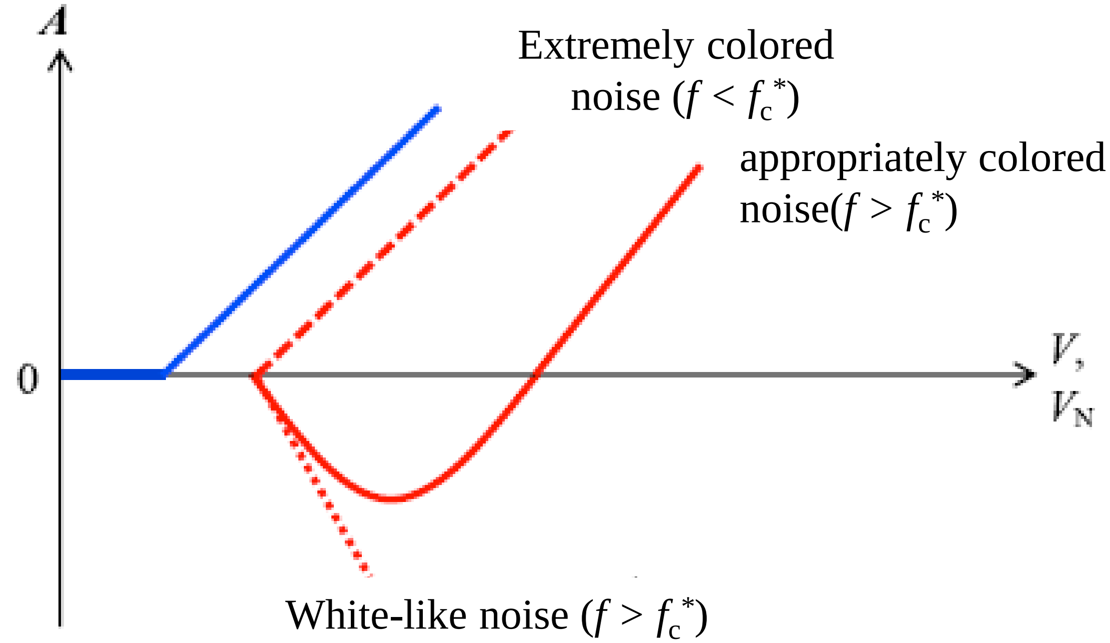
This mdoel meets the condition of "Hopf Bifurcation"
※Hopf Bifurcation : vibration of sol.

Discussions ~possible explanation of ISR~

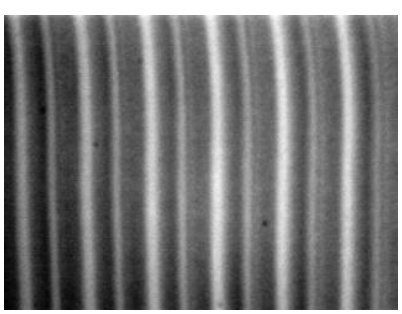
- ISR is induced by specific cutoff frequencies and intensities
  ➡ $V_N = 4 \sim 14$ V, $f_c = 500 \sim 570$ Hz
- Theoretical equation $V_c^2 = V_0^2 + bV_N^2$ should be modified.

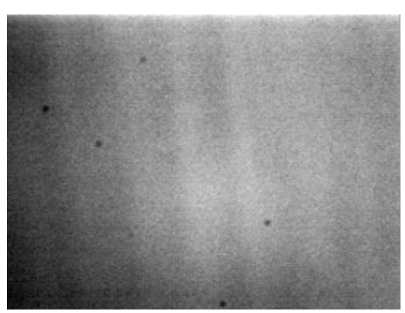
if amplitude can be supposed by a linear relation ($A = A_s + A_N$)

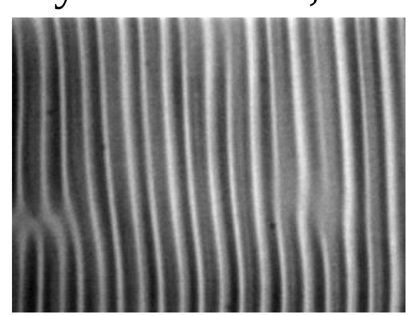




J. -H. Huh, Phys. Rev. E **94**, 052702 (2016).


(a)


(b)


(c)

2D-images (a) ~ (c) correspond to those in diagram, respectively.

Summary

Before

$V_c^2 = V_0^2 + bV_N^2$ for white noise($f_c \rightarrow \infty$, $b > 0$)

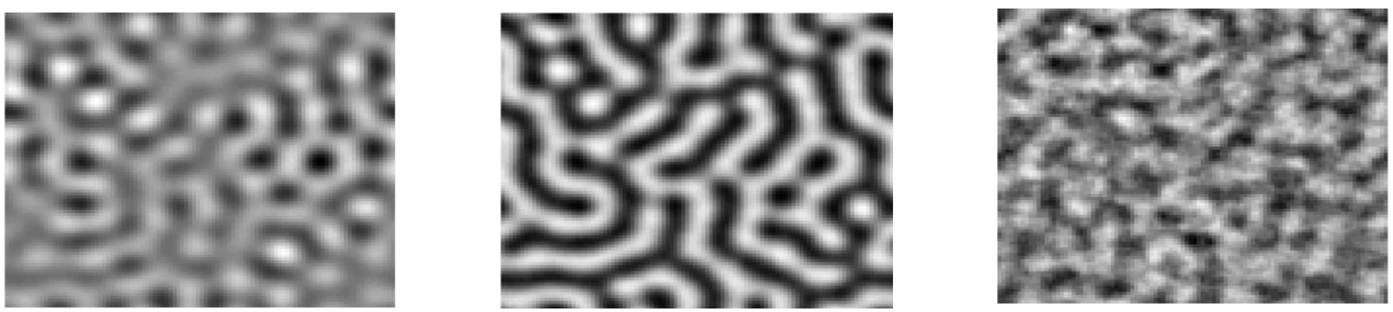
- Noise plays a role on EC threshold as a linear relationship

In this study

By employing colored noise

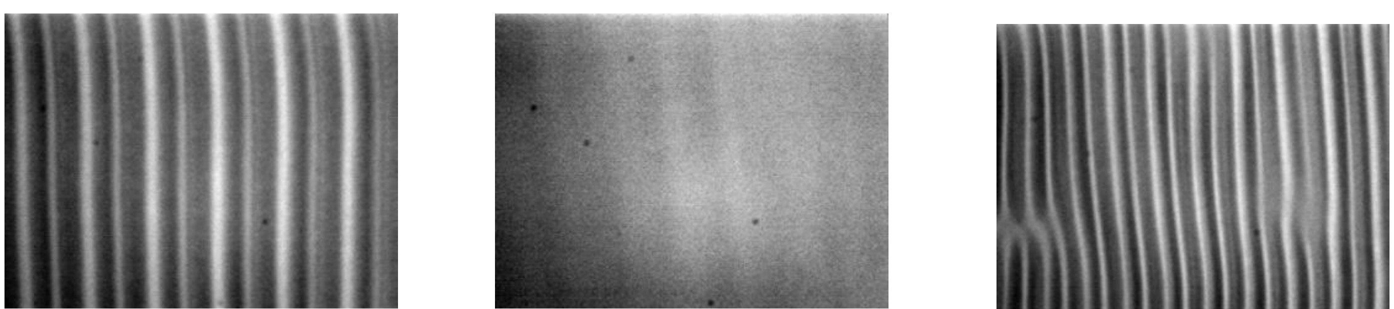
- Different behavior (ISR) is induced by appropriately colored noise
- TW requires specific condition → lower temperature

SR



J. M. G. Vilar and J. M. Rubi, PRL **78**, 2886 (1997).

ISR



In this experiment

Noise Intensity →

Acknowledgement

This work was partly supported by JSPS KAKENHI (No.15K05215)